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One-Dimensional Transport of Bosons between Weakly Linked Reservoirs

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We study a flow of ultracold bosonic atoms through a one-dimensional channel that connects two macroscopic three-dimensional reservoirs of Bose-condensed atoms via weak links implemented as potential barriers between each of the reservoirs and the channel. We consider reservoirs at equal chemical potentials so that a superflow of the quasicondensate through the channel is driven purely by a phase difference 2Φ imprinted between the reservoirs. We find that the superflow never has the standard Josephson form $\sim \sin 2\Phi$. Instead, the superflow discontinuously flips direction at $2\Phi = \pm\pi$ and has metastable branches. We show that these features are robust and not smeared by fluctuations or phase slips. We describe a possible experimental setup for observing these phenomena.

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Recent advances in trapping and manipulating ultracold gases have enabled experimental observations of a variety of new transport phenomena in quasi-one-dimensional cold atom systems [1–7], complementary to those extensively studied in condensed matter physics. Correlation effects play a crucial role in the behavior of 1D systems, and a lot of theoretical effort has been concentrated on the understanding of such effects in ultracold gases (for reviews see Refs. [8,9]).

In particular, correlation effects are responsible for a drastic modification of tunneling into a 1D channel and of a 1D flow across a single imperfection, impurity, or weak link, as has been shown in numerous theoretical [10–13] and experimental [14–17] studies of electronic transport in systems such as semiconductor quantum wires or carbon nanotubes. A geometry where these types of phenomena can be observed for ultracold atomic systems has rapidly attracted theoretical interest [18–20] and has been recently realized experimentally [4,5] by connecting 3D fermionic reservoirs via a 1D channel. A similar experiment with ultracold *bosons* would lead to the intriguing opportunity to explore coherent 1D transport focusing on features without a direct analogy in condensed matter systems.

In this Letter we study a 1D flow of degenerate ultracold bosons driven by a phase difference between two macroscopic Bose-Einstein condensates (BEC), which are weakly connected by a 1D channel via two tunneling barriers (see Fig. 1).

We demonstrate that the bosonic flow behaves drastically different to its condensed matter counterpart, i.e., an electronic flow between two bulk superconductors weakly connected by a 1D channel via Josephson junctions [21,22]. We show that qualitatively new physics emerges here. The external phase difference between the reservoirs governs the phase profile illustrated in Fig. 2: substantial

phase drops at the tunneling barriers are followed by a constant superflow of the quasicondensate through the 1D channel. Such a superflow is parametrically larger than that expected from a perturbative approach, which is appropriate for the corresponding electronic case [21] but totally fails for the bosonic superflow. Surprisingly, for an external phase difference 2Φ close to π , the phase profile turns out to be always bistable so that the superflow can spontaneously change direction (see Fig. 3). With increasing the tunneling, such a bistability spreads to all values of Φ . This would lead to jumps and hysteresis in the sawtoothlike observable superflow, making it qualitatively different from an almost sinusoidal Josephson supercurrent in the corresponding superconducting systems.

The geometry sketched in Fig. 1, required for observing these phenomena in flows of ultracold bosons, can be experimentally implemented by exploiting the versatility of potential shaping on atom chips [23]. Here we can form two bulk reservoirs weakly connected by a 1D channel and imprint an arbitrary phase difference between them, while keeping the chemical potentials equal [24].

This scenario is a starting point for experimental studies of different regimes of the bosonic superflow that we

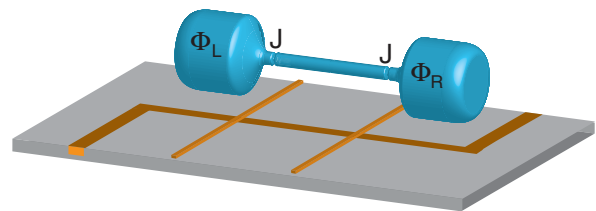


FIG. 1 (color online). A sketch of two 3D BEC reservoirs connected by a 1D channel via weak tunneling links. A simplified illustration of an atom chip creating an appropriate magnetic trapping configuration is also shown.

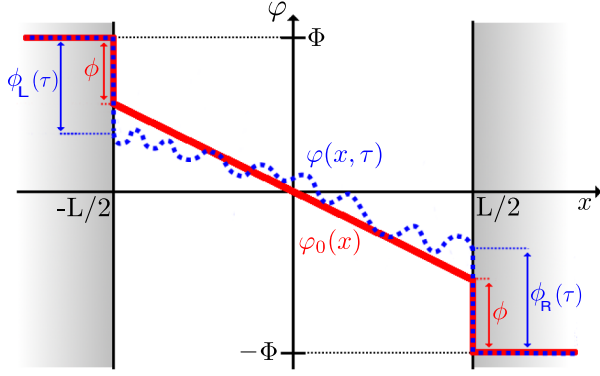


FIG. 2 (color online). Phase profile along the channel. The solid line $\varphi_0(x)$ is a typical (symmetric) configuration made up of a linear superfluid contribution and phase jumps ϕ at each tunnel barrier. The dashed line represents a fluctuation around the phase profile. Here we have chosen the phases of the BEC reservoirs as $\Phi_L = -\Phi_R \equiv \Phi$. The phase profile shown above corresponds to a phase difference of $2\Phi < \pi$.

investigate theoretically in this Letter. We will show how the results described above are obtained from a mean-field (MF) approach and prove it to be robust against fluctuations.

We consider a system comprising two bulk reservoirs, each containing a BEC, which are coupled via a 1D channel separated from the reservoirs by weak tunnelling barriers; see Fig. 1. The BEC in the left and right reservoirs is described by order parameters $\Psi_{L,R} = \sqrt{N_{L,R}} e^{i\Phi_{L,R}}$. Without loss of generality, we choose $\Phi_L = -\Phi_R \equiv \Phi$. We assume that the reservoirs have been equilibrated to the same chemical potential and thus have equal particle densities, $n_L = n_R$, so that the current through the channel is driven only by the phase difference 2Φ .

The N bosons in a 1D channel of length L form a quasicondensate described by an order parameter $\psi(x, t) = \sqrt{n + \rho(x, t)} e^{i\varphi(x, t)}$, where φ is a phase field

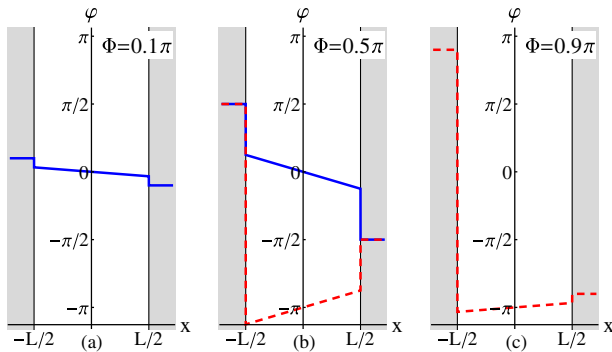


FIG. 3 (color online). The MF phase profile in the channel for $\alpha < 1$ ($J < J_c$). (a),(c) Unique symmetric (asymmetric) solutions near $\Phi = 0$ or π , respectively; (b) these two solutions become degenerate at $\Phi = \pi/2$, with one of them becoming metastable slightly above or below $\pi/2$.

and ρ denotes density fluctuations around the mean density $n = N/L$. As ρ and ϕ are canonically conjugate, the imaginary-time action describing phononlike low-energy excitations can be written via φ alone and, assuming that $L \gg \xi$, has the standard Luttinger-liquid form [25],

$$S_{LL} = \frac{K}{2\pi c} \int_0^\beta d\tau \int_{-L/2}^{L/2} dx [(\partial_\tau \varphi)^2 + c^2 (\partial_x \varphi)^2]. \quad (1)$$

Here $\xi \equiv 1/mc$ is the healing length [26], c is the sound velocity, m is the bosonic mass, $K \equiv \pi n \xi$ is the Luttinger parameter: $K \geq 1$ for bosons with a short-range repulsion, with $K = 1$ corresponding to the Tonks-Girardeau gas of hard-core bosons equivalent to the ideal Fermi gas [25]. For typical experimental situations, ξ is much larger than the distance between bosons, so that $K \gg 1$.

We model the coupling of the reservoirs to the channel by a tunneling action, assuming for simplicity [27] the tunneling energies at both barriers being equal to J ,

$$S_T = 2J \int_0^\beta d\tau [\cos \phi_R + \cos \phi_L], \quad (2)$$

where $\phi_{L,R}$ are the expected phase drops at the barriers ($x = \pm L/2$). As usual, the tunneling action is valid when the overlap of the wave functions across the barrier is small, which imposes the requirement $J \ll cK/\xi \equiv \pi n c$.

A second order in J perturbational calculation of the bosonic supercurrent gives a result divergent at $T \rightarrow 0$ for the values of K pertinent to bosonic systems. So, unlike superconducting systems [21], for which the perturbative approach is fully adequate, a nonperturbative treatment is required here.

We start our analysis with finding a nontrivial MF configuration for the model (1) and (2). The phase field $\varphi(x)$ in the channel is related to the phase drops at the barriers by the boundary conditions

$$\phi_L = \Phi - \varphi(-L/2), \quad \phi_R = \Phi + \varphi(L/2). \quad (3)$$

Then we minimize the action (1) and (2) by a stationary solution satisfying the above boundary condition,

$$\varphi_0(x) = -\phi_- - 2(\Phi - \phi_+) \frac{x}{L}, \quad \phi_\pm \equiv \frac{1}{2}(\phi_L \pm \phi_R). \quad (4)$$

It describes a constant superflow $\mathcal{I} = nv$ between the reservoirs, with a velocity $v = -2(\Phi - \phi_+)/mL$. The energy E is the sum of the supercurrent kinetic energy $\frac{1}{2}mNv^2$, which arises from the Luttinger action Eq. (1) on substituting ansatz (4), and the Josephson energy $-2J(\cos \phi_R + \cos \phi_L)$. The total dimensionless energy $\varepsilon \equiv E/J_c$ can be written via the phase drops ϕ_\pm as

$$\varepsilon = 2(\Phi - \phi_+)^2 - 4\alpha \cos \phi_+ \cos \phi_-, \quad \alpha \equiv J/J_c, \quad (5)$$

where $J_c \equiv n/mL \ll \pi nc$, so that α can vary from 0 to values $\gg 1$ within the region of applicability of the tunneling Hamiltonian, Eq. (2).

All possible MF solutions are obtained by minimizing ε with respect to ϕ_+ and ϕ_- at a fixed Φ which gives

$$\Phi - \phi_+ = \alpha \sin \phi_+ \cos \phi_-, \quad (6a)$$

$$\cos \phi_+ \sin \phi_- = 0. \quad (6b)$$

Since energy (5) is a 2π -periodic function of ϕ_- , we can restrict ourselves to two solutions of Eq. (6b), corresponding to the symmetric phase drops $\phi_- = 0$, so that $\phi_R = \phi_L = \phi_+$, and asymmetric ones $\phi_- = \pi$, so that $\phi_L = \phi_+ + \pi$. Solutions corresponding to $\cos \phi_+ = 0$ are always unstable (saddle points). For the symmetric or asymmetric branch, Eq. (6a) is reduced to

$$\Phi - \phi_+ = \pm \alpha \sin \phi_+. \quad (6c)$$

The symmetric-branch equation is almost identical to that emerging in a textbook analysis of a superconducting quantum interference device (SQUID) [28]; however, its solution has a peculiar 4π periodicity. It is the coexistence of this solution with that for the asymmetric branch that restores the correct 2π periodicity. Indeed, each of Eqs. (6c) has at least one stable solution in some interval of Φ and, remarkably, these intervals always overlap.

The MF energy is thus no longer a single-valued function of Φ . Assuming first singly connected geometry, when the external phase difference $2\Phi \in [0, 2\pi]$, we find for small Φ that the lowest energy solution of Eq. (6a), which is $\phi_+ \approx \Phi/(1+\alpha)$, belongs to the symmetric branch. An elementary analysis shows that for small α it remains stable with increasing Φ up to $\Phi = \pi/2 + \alpha$. The lowest-energy solution around $\Phi = \pi$ belongs to the asymmetric branch and remains stable down to $\Phi = \pi/2 - \alpha$. Thus, in the interval of width 2α centered at $\Phi = \pi/2$, the two solutions coexist: the symmetric solution is stable and the asymmetric solution is metastable at $\Phi < \pi/2$, with their roles reversing at $\Phi > \pi/2$, as illustrated in Fig. 3.

With α increasing, two new solutions appear at $\alpha > 1$ for both the symmetric and the asymmetric branch, but they remain unstable until α reaches $\pi/2$. At this point the two solutions coexist in the entire interval $[0, \pi]$, while new metastable solutions emerge for the asymmetric branch around $\Phi = 0$ and for the symmetric around $\Phi = \pi$. With α further increasing, new pairs of metastable solutions appear at integer multiples of $\pi/2$; see Fig. 4.

It follows from Eqs. (4) and (6a) that the superflow along the channel is $\mathcal{I} = \mp 2J \sin \phi_+$. As the sign comes from $\cos \phi_- = \pm 1$, it is easy to see that this corresponds to the sum of the Josephson currents across the barriers, $-J(\sin \phi_L + \sin \phi_R)$, as expected. What is nontrivial is the relation of this to the external phase difference 2Φ given by Eqs. (6).

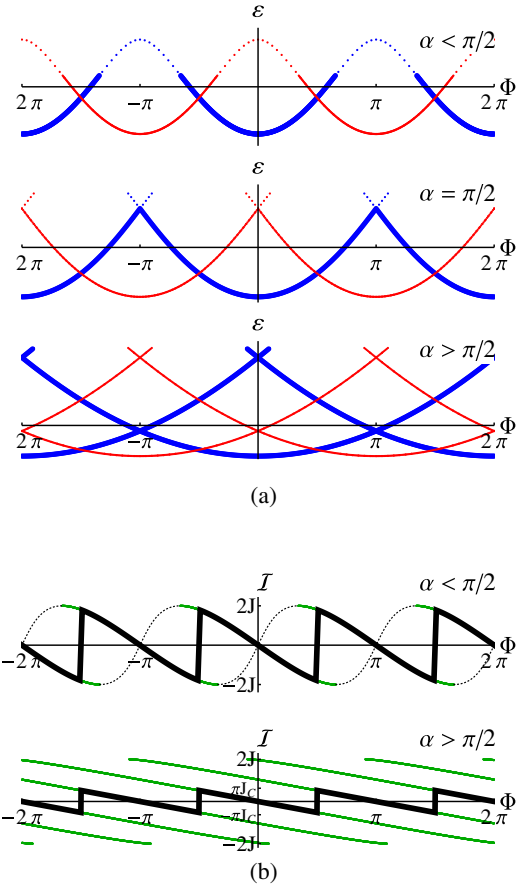


FIG. 4 (color online). (a) The MF energies (5), spread between $\varepsilon_{\min} = -4\alpha$ and $\varepsilon_{\max} = 2\alpha^2$, as functions of the external phase difference 2Φ at different values of α . Thick (thin) solid lines represent symmetric (asymmetric) stable or metastable solutions, the latter lying in the continuum of phononic excitations. (b) The dependence of the superflow on 2Φ , with thick (thin) lines representing stable (metastable) flow. Dashed lines represent unstable solutions drawn as a guide to the eye.

For small α (i.e., for $J \ll J_c$), $\phi_+ \approx \Phi \mp \alpha \sin \Phi$, so that almost the entire phase change accumulates at the Josephson barriers. The phase drops look very different for the two branches, symmetric with $\phi_- = 0$ and asymmetric with $\phi_- = \pi$. In the former case, $\phi_L = \phi_R$ by definition, while in the latter $\phi_L = \Phi + \pi$ and $\phi_R = \Phi - \pi$. This means that, e.g., near the energy minimum $\Phi = \pi$, almost the entire phase drop 2π occurs at one of the barriers. The phase profiles described by these two branches correspond to the superflows $\mp 2J \sin \Phi$, each being 4π periodic with respect to the overall phase difference 2Φ . As the symmetric branch is stable for $2\Phi < \pi + 2\alpha$ and asymmetric for $2\Phi > \pi - 2\alpha$, the correct 2π periodicity is restored by jumps between the branches which can occur anywhere in the intervals of coexistence.

With α increasing, the metastable energy solutions are reflected in the superflow, $\mathcal{I} = -J_c d\varepsilon/d(2\Phi)$, Fig. 4(b). The superflow corresponding to the lowest-energy

configuration changes from the piecewise sinusoid for $\alpha < \pi/2$ to a sawtooth function at $\alpha > \pi/2$, given by $\mathcal{I} = -2J_c\Phi$ for $\Phi \in [-\pi/2, \pi/2]$ and periodically repeated for all Φ . In the latter case, when $J \gg J_c$, the maximal possible superflow saturates at $\mathcal{I} = \pi J_c$. Such a characteristic sawtooth shape for any value of the tunneling is an inevitable consequence of the metastability. In contrast, for the case of superconductors connected by a LL channel via two Josephson junctions (corresponding in our notations to $K < 1/2$), the perturbative Josephson current has a slightly distorted sinusoidal shape [21]. It is interesting that the exact solution for the boundary case $K = 1/2$ shows a crossover from a smooth to a sawtooth shape with increasing the tunneling [22].

The existence of metastable solutions should reveal itself experimentally in hysteresis of the superflow, as we will discuss at the end of the Letter.

It is important that phonon fluctuations in the channel do not wash out essential features of the MF solutions, Eqs. (4)–(6), and remarkable that they do not result in avoided crossings in Fig. 4(a). To show this we introduce the phase fluctuations in the 1D channel, $\tilde{\varphi}(x, \tau) = \varphi(x, \tau) - \varphi_0(x)$, and at the boundaries, $\tilde{\phi}_{L,R}(\tau) = \phi_{L,R}(\tau) - \phi_{L,R}$, related by the boundary conditions $\tilde{\varphi}(\pm L/2, \tau) = \pm \tilde{\phi}_{L,R}(\tau)$. Here, $\varphi_0(x)$ and $\phi_{L,R}$ are the solutions of the MF equations (6) described above, related to the symmetric and antisymmetric combinations introduced in Eq. (4). Then, after integrating out the Gaussian fluctuations in the 1D channel, we obtain the effective action, $S = S_{\Pi} + S_{\varepsilon}$,

$$S_{\Pi} = K \int \frac{|\omega| d\omega}{2\pi^2} [|\tilde{\phi}_+(\omega)|^2 + |\tilde{\phi}_-(\omega)|^2], \quad (7a)$$

$$S_{\varepsilon} = \int d\tau \varepsilon[\phi_+(\tau), \phi_-(\tau); \Phi]. \quad (7b)$$

Here, ε is the function of $\phi_{\pm}(\tau) = \tilde{\phi}_{\pm}(\tau) + \phi_{\pm}$ and is given by Eq. (5). It plays the role of an effective “washboard” potential for the Caldeira-Legget-type action of Eq. (7a). We assumed in deriving Eq. (7a) that $\omega \gg \pi c/L$, which is the lowest phonon energy in the channel [24].

Now we perform the standard renormalization group analysis by integrating out fast modes in the fields ϕ_+ and ϕ_- , as described for completeness in the Supplemental Material [24]. This results in the renormalization group equation for the dimensionless tunneling strength α ,

$$\frac{d \ln \alpha}{d \ln b} = 1 - \frac{1}{2K}, \quad (8)$$

where b is a scaling parameter. The integration between the upper $\Lambda \sim c/\xi$ and lower $\omega_0 \sim \max\{T, c/L\}$ energy cutoffs gives the renormalized dimensionless tunneling as $\alpha(\omega_0) = \alpha_0(\Lambda/\omega_0)^{1-1/2K}$, where $\alpha_0 \equiv J/J_c$. Since the tunneling through barriers separated by $L \gg \xi$ is

uncorrelated, this is similar to the results for tunneling through a single barrier [10], as well as to the results for superconducting systems [21] in a geometry similar to that under consideration here.

A remarkable feature is that for $K \gg 1$, characteristic of ultracold bosonic systems with the healing length much bigger than the interatomic distance, α flows to larger values. This means that the washboard potential becomes more pronounced so that the fluctuations are irrelevant in the low-energy limit and the MF solution, described above, is robust. In particular, since the fluctuations do not connect different MF branches, the level crossings are not avoided and the characteristic cusps in energy, Fig. 4, and the corresponding jumps in the superflow remain. Alternatively, this can be seen using instanton techniques similar to those of Ref. [29]. Namely, the probability of an instanton connecting two degenerate configurations, as in Fig. 3(b), can be shown to be vanishingly small for $K \gg 1$.

Experimental data about the superflow can be extracted from images taken of the atomic density distribution in the channel at different times throughout the evolution of the system. The phase imprinting can be implemented in two different ways. First, we can imprint the phase difference before connecting the reservoirs, thus mapping the lowest, stable branches of the energy (Fig. 4), and measuring jumps in the superflow direction. Second, we can gradually modify the phase difference *in vivo*, with the weak link already present, thus being able to explore the metastable branches by observing a hysteretic behavior in the superflow.

Complementary direct measurements of the phase profile are possible by keeping part of the bulk BEC as a homogeneous phase reference. Then a readout can be obtained from an interference pattern between this reference and the quasicondensate in the channel [24].

In conclusion, we have demonstrated that bosonic superflow driven by a phase difference between two BEC reservoirs has spectacular features without any analogy in geometrically similar superconducting systems. The superflow, which is proportional to the first (rather than second) power of the tunneling energy, periodically flips direction and, moreover, has metastable branches, Fig. 4. The corresponding energy levels intersect, and fluctuations do not lead to avoided crossings. The bi- and multistability associated with the existence of metastable branches can only be accessed dynamically. A theoretical description of the kinetics of such a process, while going beyond the scope of this Letter, remains an interesting open question. Experimentally, the multistability can be revealed by gradually adjusting the phase difference between the reservoirs at finite tunneling.

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